ARITHMETIC AND GEOMETRIC PROGRESSION

	Arithmetic Progression	**Geometric progression**
Notation	First Term: a Common Difference: d	First Term: a Common Ratio: r
n^{th} term	$T_n = a + (n-1)d$	$u_n = ar^{n-1}$
	$n^{th}\ \text{term} = S_n - S_{n-1}$	
Sum, S_n	$S_n = \frac{n}{2}[2a + (n-1)d]$ $= \frac{n}{2}[\text{First term} + \text{Last term}]$	$S_n = \frac{a(r^n - 1)}{r - 1}\ or\ \frac{a(1-r^n)}{1-r}$
Sum to infinity, S_∞	N.A	Series is convergent for $\lvert r \rvert < 1$, $S_\infty = \frac{a}{1-r}$
Proving	$T_n - T_{n-1} = \text{constant}$ (common difference)	$\frac{U_n}{U_{n-1}} = \text{constant}$ (common ratio)
Mean	If x, y and z are consecutive terms: $y - x = z - y$	If x, y and z are consecutive terms: $\frac{y}{x} = \frac{z}{y}$

Notes:

ARITHMETIC PROGRESSION

Find the first term and the common difference of an arithmetic progression with 2^{th} term of -2 and 7^{th} term of -27. Answer: $3, -5$	Find the first term and the common difference of an arithmetic progression with 4^{th} term of -5 and 8^{th} term of 23. Answer: $-26, 7$
Find the sum of $\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, ...$ to 20 terms. Answer: 100	Find the sum of $73, 61, 49, ...$ to 18 terms. Answer: -522
Find the sum of $25, 21, 17, ..., -179$.	Find the sum of $50, 54, 58, ..., 394$. Answer: 19314

GEOMETRIC PROGRESSION

Find the common ratio and write down the n th term of the geometric progression $3, 12, 48, \ldots$. Find also the sum of the first n terms. Answer: $4, 3(4)^{n-1}, 4^n - 1$	Find the common ratio and write down the n th term of the geometric progression $8, -4, 2, \ldots$. Find also the sum of the first n terms. Answer: $-\frac{1}{2}, (-1)^{n-1} 2^{4-n}, \frac{16}{3}\left(1-\left(-\frac{1}{2}\right)^n\right)$
Find the two possible values of x if $3, x, 48$ are three consecutive terms of a geometric progression. Answer: 12 or -12	Find the two possible values of x if $-5, x, -45$ are three consecutive terms of a geometric progression. Answer: 15 or -15
Show that the geometric series of $5, \frac{5}{2}, \frac{5}{4}, \ldots$ is convergent. Find the sum to infinity. Answer: 10	Show that the geometric series of $9, -3, 1, \ldots$ is convergent. Find the sum to infinity. Answer: $\frac{27}{4}$

PROVING THE SERIES

The sum of the first n terms of a sequence is given by $S_n = 2n^2 + n$. Prove that the series in an arithmetic series and state the common difference.

Answer: common difference = 2

The sum of the first n terms of a sequence is given by $S_n = 3n - n^2$. Prove that the series in an arithmetic series and state the common difference.

Answer: Common difference = –2

The sum of the first n terms of a sequence is given by $S_n = 6 - \frac{3^{n+1}}{2^{n-1}}$. State the first term and prove that the series is a geometric series. Thus, state the common ratio.

Answers: first term = -3; common ratio = $\frac{3}{2}$

The sum of the first n terms of a sequence is given by $S_n = 5^n - 1$. State the first term and prove that the series is a geometric series. Thus, state the common ratio.

Answers: first term = 4, common ratio = 5

If a and r are both positive, prove that the series

$$\log a + \log ar + \log ar^2 + \log ar^3 + ... + \log ar^{n-1}$$

is an arithmetic series and find the sum.